BENEFIT-COST ANALYSIS AND THE MODELLING PROBLEM

Gordon A. Welty, University of Pittsburgh and Temple University

In the following essay we address a point that appears germaine to the further development of benefit-cost analysis. After establishing the nature of the technique, we discuss the concept of scope. This is a property of a predicate such as net social benefit of a project or cost-effectiveness ratio; it specifies the entities for which the predicate is relevant. Considerations of scope lead us to a discussion of the modelling problem, where the nature of the social production function becomes a concern of the benefit-cost analyst.

Benefit-cost analysis is an application of microeconomics when a comparison is made of program variants when a direct comparison of the variants in the marketplace is impossible. While it may be plausible directly to compare the efficiency of two techniques of production in the automobile industry in terms established in the marketplace, it is less plausible to make this comparison, say, for agencies in the Department of Health, Education and Welfare. In a market, the rates of exchange of commodities and services of all sorts are transformed by an adjustment process from subjectively held ratios which express personal preferences or judgments into social ratios or prices. In essence, benefit-cost analysis attempts to find surrogates or "shadows" for these prices, for the case of the bureau.

More precisely, the technique measures direct and indirect costs of each program variant. If the cost of the variant in the first year is c_1, in the second year c_2, etc. for m years, and i is the (constant) interest rate, then the present value of future costs of that variant is given by

$$c = \sum_{t=1}^{m} \frac{c}{(1+i)^t} .$$

A similar calculation is made which estimates the present value of net benefits, both direct and indirect, of each project variant as well. If the net benefit of a given variant in the first year is b_1, in the second year b_2, etc., again for m time periods, then the present value of future net benefits is given by

$$b = \sum_{t=1}^{m} \frac{b}{(1+i)^t} .$$

The net social benefit is the difference between the present values of all benefits and all costs, b-c. The decision rule prescribes that alternative where the net social benefit is greatest, while still non-negative. As we have addressed issues surrounding the estimation of costs and benefits elsewhere [13], we will merely note that benefit-cost analysis assumes that the analyst can specify the entity called a program variant and its properties.

Thus we turn from the problems of benefit-cost analysis to the prior problem of the analytical model of microeconomic theory. Clearly this model is presupposed: there is an entity which transforms inputs into outputs, the evaluation of which is given by costs and benefits. What bearing does the typical approach to benefit-cost analysis, to proceed on the basis of more or less plausible assumptions where evidence is lacking, have on this implicit theory?

Suppose we have n factors comprising a theory. For classical mechanics, n would be relatively small, including only mass, location, and time. On the other hand, the number of factors in a theory of human behavior are manifold. A general theory of price formation, for instance, would include a vast number of factors, as Professor Krelle has recently noted [3, pp. 148-149].

In making a prediction P based on the required social theory, we typically argue from hypotheses H and initial conditions C:

$$\prod_{i=1}^{n} H_i \cdot C \rightarrow P .$$

By modus tollens, we identify and rectify any inappropriate elements of the theory, vis.

$$\overline{P} \rightarrow \overline{C} \vee \sum_{i=1}^{n} \overline{H}_i .$$

The question before us is that of the identification of the erroneous element(s). If such a theory is not rectified, the analyst will continually make incorrect recommendations for action. If the backwaters of the Aswan High Dam become the breeding place for malarial parasites, this will diminish the benefit stream realized from the dam. This sort of error appears to occur in most benefit-cost analyses. Clearly, the rejection status of a hypothesis representing a particular factor (or Grünbaum's "component of the total theory"[1]) H_j is indeterminate in this case. In fact, hypotheses about all n factors may be true, if the initial conditions C are not fulfilled. Let us suppose, however, that the initial conditions are known to be fulfilled. Then,

$$\overline{P} \cdot C \rightarrow \sum_{i=1}^{n} \overline{H}_i$$

where at least one of the factors (or hypotheses) is false. Of the conditions of such a theory are the so-called scope conditions [2;9]. These conditions specify the kind of entities for which the hypotheses are to hold. For instance, a social action program might be based on a learning theory for which the scope conditions exclude children with specific learning disabilities. A factor critical to the functioning of the program is "learning ability"; it is crucial to realize that it is hypothesized that this factor is present. We will argue that the failure of such a project in a comparative benefit-cost analysis, may be due to the absence of "learning ability," due to the unfulfilled scope condition, rather than a failure of the project, curriculum, etc.

In the case mentioned, if all n factors are mutually exclusive (independent), let the (real valued) probability of hypothesis X on the basis of datum Y be $0 \leqslant Pr(X/Y) \leqslant 1$. Then,

$$Pr(\prod_{i=1}^{n} H_i/P\cdot C) =$$

$$Pr(H_1/P\cdot C)\times Pr(H_2/P\cdot C)\times \ldots$$

$$\ldots \times Pr(H_n/P\cdot C),$$

which is to say the conditional probability of a conjunction of hypotheses on given data is equal to the product of the conditional probabilities of each hypothesis on the given data. As we consider the case where the prediction was false, we have

$$Pr(\prod_{i=1}^{n} H_i/P\cdot C) = 0$$

or, alternately,

$$Pr(\prod_{i=1}^{n} H_i/\overline{P}\cdot C) = 1.$$

Now we can prove Professor Grünbaum's fundamental epistemological conclusion [1, p. 1067]. Consider, for a factor of interest H_j (Grünbaum's "main component"), that

$$Pr(H_j/\overline{P}\cdot C) = \frac{Pr(\prod_{i=1}^{n} H_i/\overline{P}\cdot C)}{\prod_{k=1}^{n-1} Pr(H_k/\overline{P}\cdot C)} ;$$

$$Pr(H_k/\overline{P}\cdot C) > 0, (k=1,2. \ . \ .n-1) .$$

The lower the conjoint conditional probability of the n-1 other factors (or auxiliary hypotheses with respect to H_j), which is to say the smaller the denominator of the right-hand expression, the higher the conditional probability of H_j in the face of a false prediction.

In the limiting case, suppose all of the auxiliary hypotheses were known true on the basis of independent supporting evidence; if they were thus conclusively verified, then

$$\prod_{k=1}^{n-1} Pr(H_k/\overline{P}\cdot C) = 1$$

and H_j is categorically refuted, which is to say $Pr(H_j/\overline{P}\cdot C) = 0$. In general,

$$1 - \prod_{k=1}^{n-1}(Pr\ H_k/\overline{P}\cdot C) > \epsilon \longrightarrow$$

$$Pr(H_j/\overline{P}\cdot C) > \epsilon.$$

$\epsilon > 0$ indicates an "irremediable inconclusiveness of the verification of [the] auxiliary component[s]" and implies "a corresponding limitation on the deducibility of the categorical falsity of the main component.[1, p. 1067]. Notice that as the threshold ϵ increases in magnitude, the possibility of falsifying H_j diminishes; hence the empirical status of the total theory vanishes in a murk of mystery.

Perhaps most frequently it is the social scientist who denies (in principle or practice) the conditions for Grünbaum's conclusion. While Professor Zeuthen, for example, explicitly accepts that "direct or indirect measurement (or the possibility of other factual testing) is a necessary condition for the avoidance of mystery," [14] others as explicitly reject the conditions. Let us examine this other viewpoint.

Of those subscribing to another viewpoint, the Austrian marginalist, Carl Menger, comes immediately to mind. Menger argued for a methodological distinction between pure economics and empirico-realistic economics. There was to be no empirical testing of the former, and of course there is little predictable in the latter [8, pp. 58-59]. More recently, Professor Machlup, in a well-known essay, proposes that the "postulate of rational action," which can be understood as "the fundamental assumption" of microeconomic theory, can be considered "as an idealization with constructs so far removed from operational concepts that contradiction by [empirical] testimony is ruled out." [4, p. 11] Thus, Machlup maintains that an economic assumption H_e postulating rational action has the conditional probability

$$Pr(H_e/\overline{P}\cdot C) = 1 .$$

Thus the rationality factor can become the basis of a scope condition: rational behavior is necessary for the applicability of economic theory.

Indeed, the probability of H_e is, for Machlup, unconditional; he goes on that

> the fundamental assumption is a resolution to proceed in the interpretation of all data of

observation as if they were the result of the postulated type of behavior. [4, p. 11]

That is to say, the assumption has a tautological implicate, as we find $Pr(H_e/\bar{P} \vee P) = 1$, which is guaranteed by the rules of valid inference alone. We must note the rather obvious point that if H_e bears any substantive weight whatever, on Grünbaum's argument the empirical content of microeconomic theory vanishes. The assumption surely does bear weight in typical economic theory, and on Machlup's argument is subject to no independent empirical verification.

But Machlup presents an alternative, by resorting to Verstehende Sozialwissenschaft. States he

the fundamental assumptions of economic theory are not subject to a requirement of independent empirical verification, but instead to a requirement of understandability in the sense in which man can understand the actions of fellowman [4, p. 17; 5, p. 487].

It is unclear what such a requirement as this implies for the social sciences, other than a methodological dualism. Indeed, Machlup later appears to qualify his dualism, citing approvingly both Max Weber and Alfred Schütz in the qualification [6, p. 291].

We find that both Weber and Schütz explicitly recognized that, quite independent of the value of Verstehen to the framing of hypotheses,

verification of subjective interpretation by comparison with the concrete course of events is, as in the case of all hypotheses, indispensible. [12, p. 97]

Thus Weber realized that empirical verification was necessary. Even in the ideal-typical case, a necessary condition for objective meaning is that the ideal type be "causally adequate."

An ideal-typical construct is said to be causally adequate when it turns out to predict what actually happens, in accord with all the rules of frequency. [11, p. 233]

We can understand "the rules of frequency" to consist for Schütz in the conventional procedures of the statistical testing of hypotheses.

Thus it appears that in the last analysis, even the foremost proponents of Verstehen as method carefully qualify themselves to methodological monism. Which returns us to Grünbaum's conclusion. To date, no one has been able to avoid the implication of vanishing empirical content of a theory, when they seek to avoid the necessity of empirical testability of the components of that theory.

Let us now return to our supposition that the initial conditions C are known to be fulfilled. If they are not known true, hence are contingencies, we must explicitly consider the probability of the initial conditions Pr(C). From Bayes' Theorem, we have for any X, Y, Z,

$$Pr(X/Y \cdot Z) = \frac{Pr(X/Z) \times Pr(Y/X \cdot Z)}{Pr(Y/Z)}.$$

By appropriate substitution,

$$Pr(H_j/\bar{P} \cdot C) = \frac{Pr(H_j/\bar{P}) \times Pr(C/H_j \cdot \bar{P})}{Pr(C/\bar{P})}.$$

Let us examine the right-hand expression. Since C may be false (unfulfilled), we must concede that $Pr(H_j/\bar{P}) > 0$. Similarly, since C may be true (fulfilled), we must concede both $Pr(C/H_j \cdot \bar{P}) > 0$ and $Pr(C/\bar{P}) > 0$. Thus,

$$Pr(C/P) \times Pr(H_j/\bar{P}) \times Pr(C/H_j \cdot \bar{P}) > 0,$$

and $Pr(H_j/\bar{P} \cdot C) > 0$ is guaranteed. In short, if we relax the supposition that initial conditions or scope conditions are known true, no hypothesis can be categorically refuted. This corresponds to our earlier suggestion that all n factors may be true. (It is readily apparent that the same conclusion follows if we relax the implicit supposition that the truth value of P is unequivocal.) As Sir Peter Medawar has emphasized,

The act of falsification is not immune to human error. [7, pp. 53-54]

On the one hand, alternatives to empirical testing are as yet a program. On the other hand, it is naive to assume that falsification alone can be a comprehensive methodological basis for the social sciences. It would appear that a more subtle view of the nature of hypothesis testing, one recognizing the potential epistemic parity of several conjoined hypotheses, must be taken by the investigator. What does all this portend for benefit-cost analysis?

An even fleeting acquaintance with benefit-cost analyses shows that they are a conglomerate of assumptions. We are immediately faced with the circumstance that initial conditions are not known to be true. Then the epistemic parity of the constituents of the theory arises.

Under such circumstances, it is likely that program variant A will be chosen over variant B because rates of transformation in social production for A appear more favorable than for B, on the basis of false assumptions about A (or B). Instead of the false assumption being rejected, the better project is rejected. This becomes particularly acute when outputs are inferred from inputs, frequently the case in bureaus. The

solution to this problem appears to lie in the careful specification of the social production function [10, pp. 56-64] for each project variant prior to benefit-cost analysis.

REFERENCES

[1] Grünbaum, Adolf, "Can We Ascertain the Falsity of a Scientific Hypothesis?" Studium Generale, Vol. 22 (1969).

[2] Hempel, Carl, Aspects of Scientific Explanation, Free Press: New York (1965).

[3] Krelle, Wilhelm, "Economic and Non-economic Factors of Price Formation," Universitas, Vol. 12 (1970).

[4] Machlup, Fritz, "The Problem of Verification in Economics," Southern Economic Journal, Vol. 22 (1955).

[5] Machlup, Fritz, "Rejoinder to a Reluctant Ultra-Empiricist," Southern Economic Journal, Vol. 22 (1956).

[6] Machlup, Fritz, "If Matter Could Talk," Morgenbesser et al (eds), Philosophy Science and Method, St. Martin's Press: New York (1969).

[7] Medawar, Peter, Induction and Intuition in Scientific Thought, Vol. 75, Memoirs of the American Philosophical Society: Philadelphia (1969).

[8] Menger, Carl, Untersuchungen über die Methode der Socialwissenschaften und der Politischen Oekonomie insbesondere, Dunker and Humblot: Leipzig (1883).

[9] Quine, Willard Van Orman, Methods of Logic, Holt, Rinehart and Winston: New York (1959).

[10] Schultze, Charles, The Politics and Economics of Public Spending, Brookings Institution: Washington (1968).

[11] Schütz, Alfred, Phenomenology of the Social World, Northwestern University Press: Evanston (1967).

[12] Weber, Max, Theory of Social and Economic Organization, Oxford University Press: New York (1947).

[13] Welty, Gordon A., "Cost-Benefit Analysis as a Guide to Action in Water Resource Projects," Highlights 1970: Congress for Recreation and Parks, National Recreation and Park Association: Washington, D. C. (1971).

[14] Zeuthen, Frederik, Economic Theory and Method, Cambridge, Massachusetts: Harvard University Press (1955).